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Special issue on global flow instability and control

Abstract This special issue is intended to provide a snapshot of current research in the area of *"Global Flow Instability and Control"*. The original papers, and to a certain extent the topic itself, are intimately linked with the series of symposia by the same name that were held in Crete, Greece, between 2001 and 2009. As members of the organizing committees of the Crete symposia, we invited all past participants to contribute, and all papers were reviewed following the strict standards of the journal. This preface gives a brief historical account of events that have shaped ideas in the field over the past decade, followed by a synopsis of the papers published herein.

Keyword Global flow instability and control

1 The Crete meetings

In July 1998, Peter Duck and Anatoly Ruban organized the EUROMECH 384 Colloquium on *"Steady and Unsteady Separated Flows"* in Manchester, with Sir James Lighthill as the keynote speaker. The proceedings of this colloquium were published as a Research Frontiers issue, volume 358, of the Philosophical Transactions of the Royal Society London journal. The preface of that issue contains the colourful hand-written transparencies of Sir James, which were bound to be the last scientific document written by this towering fluid mechanics figure. A summary version of another set of hand-written colourful drawings, corresponding to the hour-long presentation of Uwe Dallmann on flow topology [6], was included as the last chapter of the three-part paper by Theofilis, Hein and Dallmann [39] and provided a qualitative description of the first quantitative discovery, via a partial-differential-equation (PDE)-based eigenvalue problem (EVP) solution, of what is now commonly called *global mode* of a laminar separation bubble. One of the highlights in the prematurely ended scientific career of Uwe Dallmann was the enthusiastic appreciation of his flow topology work by Sir James, which is only natural, given that the first appearance of the association between flow separation and flow topology was the work of Lighthill himself in the classic textbook of Rosenhead [19].[1]

Yet another result of the EUROMECH 384 meeting was that Charbel Raffoul, attending as part of his US Air Force Liaison Officer service at the European Office of Aerospace Research and Development, urged the organization of a meeting devoted to *this new technology of partial-derivative EVP solutions* and suggested

[1] Readers following this long trail of research may find a contribution in this volume [29] and elsewhere [28] of interest.

a spot in Crete, Greece, as an appropriate venue. Of course there was nothing new regarding PDE-based EVPs at the end of last century, as such approaches had already been demonstrated in systems of vortices [27] and cylinders [15,40] almost two decades prior to that point in time.[2] What was becoming obvious at that time, though, was the fact that ever-increasing computing power permitted extending the scope of global instability analysis beyond the absolute/convective instability ideas for parallel and weakly nonparallel flows [4,13,14]; the inhomogeneity along multiple spatial directions in most fluid flow applications of interest rendered approaches based on solution of PDE-based eigenvalue problems urgently necessary. The first Crete symposium was thus born out of the need to discuss state-of-the-art and future directions primarily on global flow instability.

While the community was converging on the necessity to convene in order to assess the potential of PDE-based approaches for global flow instability and control, and a date was set for the last weekend of September 2001, the events of September 11th 2001 could not but affect that first meeting. A substantial number of confirmed delegates were unable to attend due to restricted air travel in the weeks following 9/11, and cancellation of the meeting seemed all but inevitable until, in an e-mail to the list of attendees, Israel Wygnanski wrote

> "*...we in Israel are under constant threat of attack, but manage to live more-or-less normal lives. If we back off our plan to meet, the terrorists will have won the battle. Let us all go to Crete.*"

Wygy's inspiring message proved decisive in reversing the trend of last-minute cancellations and helped make the *Crete meeting* a reality. Continued financial support from the European Office of Aerospace Research and Development and the Air Force Office of Scientific Research permitted the continuation of this series with 4 meetings having been held through 2009, while a 5th meeting is planned for 2012. Summary documents, abstracts of presentations and transcripts of the discussion sessions are available from the authors upon request.

2 Progress made in the last decade

A key objective of the Crete meetings has been the development of a common language that allows a precise demarcation of new contributions, and which, in turn, will best exploit the synergy between the (sometimes independent) stability and control communities. The need, often overlooked in recent literature, has arisen for careful and consistent use of terminology used to describe flow stability concepts, especially as faster computers allow for far more complex analyses than traditional (parallel) flow stability analysis allowed. Aside from leading to confusion within the community (which was immediately evident in the group discussions of the Crete I meeting already, and persists today!), there is the more dangerous possibility of misinterpretation of results, or failure to identify instabilities by employing an incorrect stability paradigm. Moreover, experimental and computational apparatuses can be *contaminated* by global instabilities that may be artifacts of the confinement of a wind tunnel or computational domain.

The *basic* (or base) state term is reserved for flows that represent exact solutions of the Navier-Stokes equations, regardless of their observability in nature. That is to say, they may represent *unstable* equilibrium (steady or unsteady) solutions, and/or they may invoke idealizations of boundary conditions (i.e. two-dimensionality) that can only be achieved approximately in the laboratory. Examples include steady laminar or time-periodic flows (e.g. the 1-D Poiseuille/Couette profiles or the 2-D Kármán wake of a circular cylinder at $Re \geq 41$). Rapidly increasing computational resources pave the way for an increasing complex repertoire of such flows to be thoroughly analysed, and bifurcations categorized. This has been coupled to advances in numerical algorithms for bifurcation analysis of such flows. Unfortunately, a majority of applications, including most flow control experiments, are at far higher Reynolds numbers than such analysis allows, and indeed, in those cases only the *mean* flow is available from experiment. While linearized solutions to turbulent mean flows have been widely considered, such application requires a scale separation between the energetic turbulent fluctuations and the disturbances, and the modes associated with these flows do not necessarily represent true instabilities, regardless of their rate of transient or asymptotic growth. A simple example demonstrating the difficulties presented by scale separation arguments is that a rigorous Floquet stability analysis of a periodic flow gives results that are quite different from the stability of the time-average or various snapshots of the same flow.

To maintain analytical or computational tractability, traditional stability theory has focused on classifying base flows that are independent of two out of three spatial coordinates; then disturbances have a wave structure in the two homogeneous directions. The most important examples are of course boundary and free-shear layers, for which the slowly varying streamwise changes in the basic flow are neglected or treated as a small

[2] References [37,38] provide a reasonably full account of the topic.

parameter in an asymptotic expansion. It is long established that the basic state of parallel linear theory may be either *convectively* or *absolutely* (un)stable, when a disturbance propagates, respectively, downstream only or both down- and upstream from the point of its introduction into the flow. In both cases, the flow is said to be *locally* (un)stable, on account of the local character of the underlying 1-D basic state. If the latter assumption is relaxed and a basic state which depends strongly on one and mildly on a second spatial coordinate is considered, the flow is said to be either convectively or globally unstable, depending on whether local absolute instabilities give rise to a global oscillation frequency (self-sustaining oscillations). It should also be recognized that other means of feedback (acoustic, actuation, etc.) could also give rise to globally unstable flows, even if the underlying basic flow is locally convectively unstable. The absolute/convective theory and its recent nonlinear extensions has proved a powerful tool for understanding frequency selection criteria for weakly nonparallel flows, and the prevailing use of the term *global* in the stability literature concerns such flows.

But, especially in recent years, the term *global* instability also describes unstable disturbances to two- and three-dimensional basic flows, including application of the full machinery of linear stability analysis to basic flows that are only weakly nonparallel. Clearly the answer to whether a given base flow is unstable or not (and if it is stable, whether it is asymptotically stable) should be independent of the method of solution of the EVP, but it is not obvious that this is true for many of the weakly non-parallel basic states that have been examined with the local analysis, and, indeed, many of these basic flow are only approximately solutions of the Navier-Stokes equations in the first place. To avoid confusion, discussions at the Crete meetings led to a proposal to classify the analyses as either *Global-A* (or uni-global), for instability of weakly nonparallel basic flows, and *Global-B* (or *BiGlobal*) and *Global-C* (or *TriGlobal*) to describe instability of two- and three-dimensional basic states, respectively. These definitions have the merit of disambiguation and have therefore been adopted by a subset of the community, but it is important to point out that they refer to the method of solution of the EVP, rather than a classification of the instability associated with a given basic flow. In this context, it is also worth pointing out that the same terminology problems arise in analyses of *transient growth*. Such analyses, which attempt to determine the amplification factor for disturbances to a basic state regardless of its asymptotic stability, can, of course, be performed locally or globally. The aforementioned effects of boundary conditions and approximations to the basic flow are also a key issue and can have a dramatic impact on the asymptotic stability and implied transient growth rates.

There are deep connections between stability, modelling and control of flows [5]. First and most obvious is the desire to actuate a flow at a point where the flow is most sensitive, or receptive, to small disturbances. Stability analysis provides a clear means of identifying such points, as well as establishing the expected amplification of the (hopefully) small disturbances added to the flow. Clear examples of this have been understood for many years; from a theoretical point of view, the seminal work of Hill [11] provided a major impulse to this end. A prime goal of current work is therefore the identification of key global instability information (critical Reynolds number, frequency of different modes, viscous or inviscid nature of the latter) which can be used to extend the paradigm to more complex, industrially relevant flows. A great success of both (uni)global flow instability which follows the absolute/convective ideas and global analysis based on numerical solution of direct and adjoint partial-derivative eigenvalue problems [11] has been their potential to identify points/areas in the flow field which determine global instability of the entire flow. The region around this point is expected to play a key role in efficient flow control strategies, as emphatically demonstrated in the work of [10], originally presented in the Crete II meeting, in 2003.

A related area that has grown significantly since the first Crete meeting is the ability to describe the dynamics of the flow using a (relatively) simple, reduced-order model that may be, in turn, used to design a controller for the flow. Control of nonlinear and large systems remains a challenging discipline and, to a certain extent, issues of control design are distinct from the main topics of this volume. However, at the heart of any control problem is an understanding and description of the natural dynamics and instabilities of the system, and a model to describe them with sufficient fidelity and of sufficiently low order to be amenable to the tools of modern control design. Early work [12] focused on the proper orthogonal decomposition/Galerkin projection approach for obtaining reduced-order models, but a variety of issues, ranging from the fragility of the models in changing flow conditions to difficulties with implementing the effects of local actuation on the global modal structure, have prompted revisions, extensions, and alternatives to these techniques. One approach is to supplement the POD-based subspace with eigenfunctions from stability analysis, shift modes, scheduled basis sets, etc. [25]. Other approaches seek alternative subspaces; examples, which include work presented in this volume, include *balanced truncation* [30] and the *dynamic mode decomposition* (DMD, alternatively *Koopman modes*) [33].

3 Contributions to the present volume

This collection of original research papers testifies to the large progress that the areas of *"Global Flow Instability and Control"* have experienced in the last decade. As discussed above, the range and complexity of basic flows that are the target of stability and control continue to expand, and this volume includes a wide range of applications that includes aircraft wake vortices [7], jet in crossflow [31], impinging jet [24], pulsatile flow in a stenotic pipe [23], a low-pressure turbine blade [34] and separated flows on flat plates, wings and bluff bodies [8,16,29,35]. Refinements and extensions to the theory for the classical flat-plate boundary layer [1,21], corner flow [20] and Jeffery-Hamel wedge flow [36] are also included. While the bulk of the analyses address incompressible flows, stability analysis for the swept airfoil boundary layer [18] and flow over roughness elements [2] show that the analyses may be fruitfully applied to compressible and hypersonic flows, respectively.

The contributions may also be broadly classified into either of the "stability" and "control" areas. For stability (including transient growth), the above flows include two-dimensional, quasi-three-dimensional (two inhomogeneous and one weakly nonparallel) and fully three-dimensional basic states have been analysed, employing viscous or inviscid flow instability theory, as appropriate. The instability analyses all employ partial-derivative initial- and eigenvalue problem solutions, including PSE-3D, either on their own, or in combination with longer-established methodologies, such as triple-deck theory [20] or direct numerical simulation [17], or more modern space-time CESE [3] numerical methods. Mostly iterative eigenspectrum computation methods [9] have been employed, although analyses based on direct full-spectrum computations [27] have also been presented. Evidence is presented that the imposition of harmonic dependence of perturbations in one spatial direction, as done in the context of BiGlobal analysis, may inhibit growth of perturbations in swept-wing [18] and trailing-vortex flows [7]. Relaxation of the homogeneity assumption and adoption of the PSE-3D concept in the former and a rational multiple-scales expansion in the latter application permits recovering results in excellent agreement with experiment. Composite flow fields constructed by linear superposition of steady two-dimensional separated basic flow on an airfoil and the corresponding leading stationary three-dimensional unstable global mode reveals that the origin of the stall-cells structures on the airfoil is modal linear amplification of the leading global flow eigenmode [29]. Transient growth studies are performed to identify optimal perturbations in complex biologically and industrially relevant flows, such as an adverse-pressure-gradient separated flow [1], the wake of a low-pressure turbine [34] and a stenotic geometry with a physiological waveform [23]. Finally, the signalling problem, i.e. the spatial response of a system to localized harmonic forcing, which is traditionally associated with convectively unstable and/or absolutely stable systems, has been demonstrated for absolutely unstable systems [26].

Turning to control, several contributions deal with the decomposition of flow data into dynamically relevant modes, and the subsequent generation of control-oriented, reduced-order models. The Dynamic Mode Decomposition (DMD), which uses experimental or numerical data to reconstruct a low-dimensional inter-snapshot map, which is subsequently utilized to break down a physical process in dynamically relevant modes and coherent structures, is examined in the context of time-resolved PIV measurements of forced and unforced jets [32]. The Eigensystem Realization Algorithm (ERA) is shown to produce equivalent reduced-order models as the balanced POD technique [22], which is in turn a computationally tractable simplification of the more general balanced truncation technique, but without recourse to data from solutions to *adjoint* equations, and thus applicable to experimental data. The relationship between global linear eigenmodes and the POD modes of a jet in crossflow is examined [32]. Adjoint global modes and sensitivity analysis are used to identify the regions of an impinging jet suitable for different kinds of actuation, and (passive) control of the flow is observed when a small airfoil (lift force) is placed in the flow in the specific locations identified by the analysis [24]. Two papers exploit reduced-order models based on POD and its generalizations to attenuate TS waves via localized wall actuation [21] and to control disturbances to a model separated boundary layer [8]; a common theme in both papers is to develop alternatives to the traditional POD/Galerkin approach that result in more *robust* reduced-order models that can represent the effects of localized actuation on the global set of modes. Finally, two papers develop heuristic controllers to modify vortex shedding in the wake of a bluff body or airfoil at high angle of attack. These exploit the concept of a phase-lock-loop, which is able to track the slowly varying, instantaneous phase of the instability which is subsequently fed back to attenuate [35], or enhance [16] shedding.

4 Summary

Flow stability analysis remains a critical research area for understanding the behaviour of a variety of industrially, biologically and environmentally relevant flows. Indeed, a variety of new computational and theoretical approaches have added considerably to the repertoire of (increasingly complex) flows whose bifurcations we hope to understand. Meanwhile, flow control has benefited from rapid advances in hardware (actuation and sensing), but increasingly also from a better understanding of the underlying dynamics of the unsteady and often unstable flows which are to be controlled. Flow stability analysis has a key role to play in the latter, both in the identification of locations within the flow that can control global instabilities (and where the flow is receptive to actuation), and also in the generation of models that can be used to devise control laws. Future challenges include the use of computational and experimental data, together with systematic model-reduction techniques, to develop nonlinear flow models that can be used for bifurcation analysis and control design in increasingly complex, three-dimensional turbulent regimes.

Acknowledgments The Crete symposia have been sponsored by the European Office of Aerospace Research and Development, Air Force Office of Scientific Research, Air Force Material Command, USAF, under Grants to *nu modelling s.l.*, monitored by Dr. Charbel Raffoul (1997–2002), Mr. Wayne Donaldson (2002–2005) and Dr. Surya Surampudi (2005–2010). The views and conclusions contained herein are those of the author and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the Air Force Office of Scientific Research or the US Government.

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